"USE OF PRESSURE, PRESSURE-SQUARED OR PSEUDO-PRESSURE IN THE ANALYSIS OF TRANSIENT PRESSURE DRAWDOWN DATA FROM GAS WELLS"

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this article begins on the next page
Use of Pressure, Pressure-Squared or Pseudo-Pressure in the Analysis of Transient Pressure Drawdown Data from Gas Wells

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Abstract
In the analysis of transient pressure drawdown data, it is possible to plot the data in terms of pressure, pressure-squared or pseudo-pressure $- 2 \int p/\mu dV$. In this paper, we compare these three approaches and provide some guidelines for the use of each. The use of pseudo-pressure is found to be the only reliable method when drawdowns are small and well testing is not performed. The pressure-squared approach is found to be better than the other two, when these two are different. Our conclusions are based on the comparison of analytical solutions with numerical solutions for several typical gas reservoirs in Alberta.

Introduction
In the reservoir engineering literature, the flow of gas is invariably analyzed by using one of three variables — namely, the pressure, the pressure-squared or the pseudo-pressure. There is no clear evidence concerning the conditions of flow under which the use of some of these variables is admissible.

Rawlins and Shellhardt (1935) found empirically that stabilized gas flow could be described in terms of pressure-squared by the well-known deliverability equation for gas in the reservoir could be represented by the diffusivity equation expressed in terms of pressure-squared (instead of pressure, as in the case of oil flow).

Matthews and Russell (1967), in their discussion of pressure build-up and analysis, suggested that using the pressure-squared was unnecessary and that satisfactory results were being obtained by simply using the pressure as the variable in the diffusivity equation.

Al-Hussainy and Ramey (1966) proposed yet a third variable, originally termed the "real gas potential", but later referred to as the "real gas pseudo-pressure." We refer to this simply as the "pseudo-pressure." There are then three well-known approaches for the plotting and analysis of well data: pressure, pressure-squared and pseudo-pressure. To complicate matters, some pseudo-pressure data may be analyzed by using any one of these three variables, and give the same answer. Other data will give a different answer for each of the variables. Moreover, deliverability tests on gas wells are usually analyzed using the pressure-squared, whereas drill-stem tests or buildup tests are often analyzed using the pressure. This gives rise to the question of consistency of results.

Further complication and confusion results because some of the parameters that must be constant for the application of the analytical...
solution are actually dependent on pressure. What pressures should these variable parameters be evaluated at? A clear answer to this question is also not available in the literature.

The purpose of this publication is to clarify the relationship between the three systems of analysis, and to indicate clearly when they are consistent with each other and when they are not. We will also consider the question: At what constant (or average) conditions should the variable parameters be evaluated? This study is restricted to constant-rate production from infinite-acting reservoirs. A study of finite reservoirs and other types of tests will be reported at a later time.

**Theory**

It is instructive to first review the derivation of the various forms of the diffusivity equation. The continuity equation, the equation of state for a real gas, and Darcy's law (in the one-dimensional, radially-cylindrical form) may be combined to give (Al-Hussainy and Ramey, 1966):

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) = \frac{k}{\mu} \frac{\partial}{\partial t} \left( \frac{p}{2} \right)
\]

So far, Equation (2) involves the following assumptions only:

![FIGURE 1 — Pressure drawdown calculated by different methods for Test No. 8 of Table 1.](image)

**TABLE 1 — Reservoir and Gas Data for the Comparison of the p, p^2 and \( \psi \) Approaches**

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isothermal conditions, negligible gravitational effects (constant porosity and permeability), and single-phase one-dimensional Darcy flow (non-Darcy flow can be taken into account by expanding the usual definition of skin factor to include inertial-turbulent effects).

PSEUDO-PRESSURE

Without having to make any further assumptions, and by defining the pseudo-pressure as follows:

\[ \psi = 2 \int_{p_0}^{p} \frac{dZ}{\mu Z} \]  

and noting that the isothermal compressibility of a gas is given by:

\[ c = -\frac{1}{\rho} \frac{d\rho}{dT} \]  

Equations (3) and (4) may be substituted in Equation (2) and manipulated to give:

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \psi}{\partial r} \right) = \frac{\Phi \mu c}{k} \frac{\partial^2 \psi}{\partial t^2} \]  

This is the diffusivity equation in terms of the pseudo-pressure (Al-Hussainy and Ramey, 1966).

PRESSURE-SQUARED

There are three sets of assumptions possible, any one of which results in the pressure-squared form of the diffusivity equation. These assumptions are in addition to those required for the development of Equation (2).

Set I:

Ideal gas behaviour is assumed \((Z=1)\) and gas viscosity is assumed to be independent of pressure. Under these conditions, Equation (2) becomes:

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \psi}{\partial r} \right) = \frac{\Phi \mu c}{k} \frac{\partial^2 \psi}{\partial t^2} \]  

This is similar to the form of the equation used by Jenkins and Aronofsky (1953), and Aronofsky and Jenkins (1954).

Set II:

The product \(\mu Z\) is assumed to be constant. Equation (2) then reduces to:

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \psi}{\partial r} \right) = \frac{\Phi \mu c}{k} \frac{\partial^2 \psi}{\partial t^2} \]  

Set III:

Pressure gradients are assumed to be small, i.e. terms like \((\frac{\partial\psi}{\partial T})^2 \rightarrow 0\).

Equation (2) again reduces to Equation (7).

PRESSURE

If in Equation (2) the quantity \(v/\mu Z\) is assumed constant (compare to assumption in Set II), the pressure form of the diffusivity equation results as below:

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \psi}{\partial r} \right) = \frac{\Phi \mu c}{k} \frac{\partial^2 \psi}{\partial t^2} \]  

Any one of Equations (5), (7), or (8) can be written in the following form:

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \psi}{\partial r} \right) = \frac{1}{K} \frac{\partial \psi}{\partial t} \]  

where \(\Phi\) represents either p, \(p^2\) or \(\psi\) and \(K\) represents \(k/\mu c\).

DIMENSIONLESS EQUATION

The dimensionless form of Equation (9) is:

\[ \frac{1}{r_0} \frac{\partial}{\partial r_0} \left( r_0 \frac{\partial \psi_0}{\partial r_0} \right) = \frac{\partial \psi_0}{\partial t_0} \]  

where

\[ r_0 = r/\kappa \]  

\[ t_0 = 2.637 \times 10^{-4} \frac{1}{\Phi \mu c} \]  

and the definition of \(\Delta p_0\) depends on the approach used.

For the pressure treatment the definition is:

\[ \Delta p_0 = \frac{r_1 - r_0}{7.085 \times 10^{10} T q_r} \frac{\mu Z}{\rho} \]  

For the pressure-squared treatment the definition is:

\[ \Delta p_0 = \frac{r_1^2 - r_0^2}{1.417 \times 10^{4} \frac{1}{\kappa h} \frac{\mu Z}{\rho}} \]  

For the pseudo-pressure treatment the definition is:

\[ \Delta p_0 = \frac{\Phi \psi - \Phi \psi_0}{1.417 \times 10^{4} \frac{1}{\kappa h} \frac{\mu Z}{\rho}} \]  

The parameters \((1/\mu c)\) in Equation (11), \((\mu Z/p)\) in Equation (12) and \((\mu Z)\) in Equation (13) must be assumed constant at some pressure, if the analytical solution.

FIGURE 2 — Pressure drawdown calculated by different methods for Test No. 11 of Table 1.
is to be used for the analysis of well data. If these variables cannot be assumed constant at initial conditions then, strictly speaking, the analytical solution is not valid. If we must use the analytical solution, then we must also ask the question: What is the best approach to calculating average values of these parameters?

**SOLUTION OF RADIAL FLOW EQUATION**

The analytical solution of Equation (10) for various boundary conditions is available [e.g., Matthews and Russell, 1967]. For our discussion, we will only consider radial flow in an infinite-acting reservoir. For this case, the solution of the flow equation at the well (sandface) is given by:

\[
\Delta p_w = -\frac{1}{2} \ln \left(\frac{1}{4t_0}\right) \quad (15)
\]

which may be approximated for \( t_0 > 25 \) by

\[
\Delta p_w = \frac{1}{2} \ln (\ln (t_0) + 0.009) \quad (16)
\]

It is possible to solve Equation (2) by numerical methods to any desired accuracy and to use the solution thus obtained to test various methods of analysis.

**COMPARISON OF VARIOUS METHODS**

Table 1 provides data for 24 typical gas wells from reservoirs in Alberta. The initial pressures for these reservoirs and the pressures at the well, after the flowing period specified in Table 1, are given in Table 2. Columns (3) to (10) provide eight different sets of pressures calculated by eight different methods. The methods used are also described in Table 2. They include the calculation of the various parameters A, B, and \( \psi \) initial and averaged conditions for all three approaches. Also considered are different methods of averaging gas properties. For many of the cases, particularly those involving low drawdowns, the differences in calculated pressures are small. However, in some of the cases the differences in computed pressure are very large; entries numbered 8, 11 and 16 are examples of this behaviour.

Figures 1, 2 and 3 compare pressure drawdowns by different methods for Test Nos. 1, 11 and 16 respectively. Also included on these figures are the numerical solutions. A careful examination of all of these results reveals that if average properties are used all three approaches yield results which are

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<th>( p^2 ) Approach</th>
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Column (2) gives the predicted drawdown values = \( (p_i - p)/p \), where \( p \) is from column (3). *Calculated pressure is negative.

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quite close to the numerical solution. However, when properties must be assumed constant at initial conditions and when the drawdown is high, only the pseudo-pressure approach is reasonable. In many cases, calculations by the pressure or pressure-squared approaches with variable parameters at initial conditions give unreasonably high pressure drawdowns. In the three cases considered in detail, the pressure-squared approach with properties at initial conditions always resulted in solutions more accurate than those obtained by the pressure approach with properties at initial conditions.

**Pressure-Squared Approach Range of Validity Of Pressure or**

Al-Hussainy and Ramey (1966) have shown that, in the pseudo-pressure case, \( \mu Z \) evaluated at initial conditions resulted in an excellent match with their numerical solution. We also find that this approach gives better results than the other two approaches when the coefficients are evaluated at initial conditions. However, for some of the cases considered, the pseudo-pressure approach with \( \mu Z \) at initial conditions differs from the more accurate numerical solution by a substantial amount (see Fig. 1). Nevertheless, the pseudo-pressure form of the diffusivity equation [Equation (5)] is the most rigorous and may be used as a yardstick when investigating the validity of the pressure-squared form or the pressure form.

Wattenbarger and Ramey (1968) plotted \( \mu Z \) versus \( p \) for a specifically chosen natural gas. They showed that, at low pressures, where \( \mu Z \) is nearly constant, the pseudo-pressure equation becomes the same as the pressure-squared equation, with \( \mu Z \) and \( c \) taken at initial conditions. They also showed that at higher pressures, where \( \mu Z \) is nearly constant, the pseudo-pressure equation becomes the same as the pressure equation, with \( \mu Z \) and \( c \) taken at initial conditions. Unfortunately, the relationship between \( \mu Z \) and \( p \) for other gases and at other temperatures is quite different. In Figure 4, we show \( (\mu Z) \) for natural gases as a function of \( p \) for various values of \( T \). Values of \( (\mu Z) \) were obtained from the correlation of Carr et al. (1954). Values of \( Z \) were obtained from a computerized version (Dranchuk et al., 1974) of the Standing and Katz (1942) correlation.

Clearly, the simple generalization that at low pressures the pressure-squared equation is valid and at high pressures the pressure equation is more applicable is not acceptable (see Fig. 4).

Even though different sets of assumptions were used in deriving the pressure-squared or pressure equations, it is not usually obvious, in practice, which one, if any, of these sets of assumptions applies to a particular situation. Accordingly, a more practical approach to the problem was sought. This has resulted in the development of a practical method for choosing between pressure and pressure-squared as the variable to be used for analysis. Let us consider Equation (5) in terms of \( \psi \), and note that it reduces to Equation (8) in terms of \( p \) if the plot of \( \psi \) versus \( p \) is a straight line (not necessarily passing through the origin), i.e.,

\[
\psi = k_1 p + k_2 \quad \ldots \ldots \ldots (17)
\]

Substituting Equation (17) in Equation (5) results in Equation (8), the \( p \) form of the diffusivity equation.

Similarly, if the \( \psi \) versus \( p^2 \) plot is a straight line,

\[
\psi = k_3 p^2 + k_4 \quad \ldots \ldots \ldots (18)
\]

Substituting Equation (18) in Equation (5) results in Equation (7), the \( p^2 \) form of the diffusivity equation.

If, for a particular gas, \( \psi \) is plotted versus \( p \) (or \( p^2 \)) a curve as shown in Figure 5 is obtained — curve I. It is obvious that any part of that curve, covering a limited range of \( p \) values, may be considered to be a straight line. In that range, the \( p \) and \( \psi \) approaches will give comparable answers.

If for the same gas \( \psi \) is plotted versus \( p^2 \) (or \( p^3 \)), another curve is obtained — curve II — any segment of which, covering a limited range of \( p^2 \) values, may be considered as a...
straight line. In the range of that segment that \( p^2 \) and \( \psi \) approaches will give comparable answers.

However, it is immediately obvious that provided the range of pressures being investigated is sufficiently small, both the \( p \) and \( p^2 \) curves may be considered to be straight and either approach will give the correct answer, irrespective of whether the gas is at high or low pressure.

This analysis can be easily extended to all natural gases, at all pressures and temperatures of interest to a reservoir engineer, by plotting the \( \psi \) and \( p^2 \) plots in dimensionless terms. Let us define a reduced pseudo-pressure as

\[
\psi_r = \frac{\psi}{\mu Z} = \frac{\psi}{\mu Z} \quad \text{and} \quad \psi_p = \frac{\mu Z}{2p^2} \quad \text{ ...... (19)}
\]

\( \psi \) can then be evaluated at various values of \( p \) and \( T \), using the generalized correlations of Carr et al. (1965) for \( \mu/\mu_1 \), and of Standing and Katz (1942) for \( Z \). A plot of \( \psi \) versus \( p \) at various values of \( T \) is shown in Figure 6. A plot of \( \psi \) versus \( p^2 \) at various values of \( T \) is shown in Figure 7. Note that the lower limit used in Equation (19) is different from that used by Al-Hussainy and Ramey (1966). We find the lower limit of zero to be more convenient.

For any natural gas, therefore, the process of choosing between \( p \), \( p^2 \) and \( \psi \) in pressure analysis work may be summarized in the following three-step procedure.

1. For the appropriate \( T_r \), select from Figure 6 the relevant \( \psi \) versus \( p \) plot (curve I), and from Figure 7 the relevant \( \psi \) versus \( p^2 \) plot (curve II).
2. Determine the initial and final pressures of interest and convert them to their equivalent \( p \) values.
3. Mark these \( p \) values on each of the curves, and in this interval choose that curve which approximates a straight line better.

If curve I is chosen, the \( p \) approach is preferable to \( p^2 \).
If curve II is chosen, the \( p^2 \) approach is preferable to \( p \).
If both give acceptable straight lines, then either \( p \) or \( p^2 \) may be used with good results.
If neither curve may be considered to be reasonably straight in that range of pressures, then neither \( p \) nor \( p^2 \) is acceptable and \( \psi \) should be used.

The last question to be answered is: What values of \( \mu \), \( c \) and \( Z \) are to be used in the \( p \) or \( p^2 \) treatments.

In the \( p \) case,

\[
\frac{\partial \psi}{\partial p} = k_i = \frac{2 \gamma}{\mu Z} \quad \text{ ...... (20)}
\]

FIGURE 4 — Variation of \((\mu/\mu_1)Z\) with pressure at different temperatures.

FIGURE 5 — Variation of reduced pseudo-pressure with reduced temperature and pressure \((T_r = 1.7; 0 \leq p_r \leq 15)\).
\[ \frac{2p}{\partial p^2} = k_1 = \frac{2p}{\mu Z} \frac{1}{2} = \frac{1}{\mu Z} \]  

(21)

e.g., \( \mu Z/p \) is constant in the range where the \( \psi \)-p curve is straight. Therefore, any value of \( \mu Z/p \) in that range will be acceptable.

In the \( p^2 \) case,

\[ \frac{2p}{\partial p^2} = k_1 = \frac{2p}{\mu Z} \frac{1}{2} = \frac{1}{\mu Z} \]  

(21)

e.g., \( \mu Z \) is constant in the range where the \( \psi \)-\( p^2 \) curve is straight. Therefore, any value of \( \mu Z \) in that range will be acceptable.

As an example of the application of Figures 6 and 7, let us consider the data of Test No. 8. From Figure 1, we see that neither the \( p \) nor \( p^2 \) approach (with properties at initial pressure) is acceptable, but the \( p^2 \) approach is slightly better. This is confirmed if we look at the \( \psi \) vs \( p \) and \( \psi \) vs \( p^2 \) curves for \( T_r = 1.7 \) and \( p \), from 1.5 to 5 in Figure 5.

Conclusions

1. Calculation of well pressure from the analytical solution is more accurate if the average properties, rather than properties at initial conditions, are used. This is true regardless of the approach (\( p \), \( p^2 \) or \( \psi \)) used. The differences between the three approaches with average properties and the numerical solution are small.

2. When the arithmetic mean is used, the \( p \) or \( p^2 \) approaches become identical as far as the calculation of well pressure from the analytical solution is concerned.

3. The method of averaging properties does not have a large influence on the solution. A simple arithmetic mean is usually satisfactory.

4. In the use of analytical solutions for infinite reservoirs, it is necessary to be able to assume that \( \mu Z \) is constant regardless of the approach used. In addition, for the \( p^2 \) approach \( \mu Z \) must be assumed constant and for the \( p \) approach \( \mu Z/p \) must be assumed constant.

Large differences may result in calculated well pressures between the three approaches when initial values are used for these variables. For high-drawdown wells (where these differences are very large), the \( \psi \) approach is usually most accurate, followed by the \( p^2 \) approach. The \( p \) approach is least accurate in most cases where drawdowns are high.

5. The \( \psi \) approach with \( \mu Z \) at initial pressure results in reasonably accurate well pressures from the analytical solution. If fluid properties are to be assumed constant at their initial values, then this is
the only reasonable approach when large drawdowns (50% or so) are possible.

6. When drawdowns are small (15% or so), any of the three approaches \( \left( \frac{p}{p_i}, \frac{p^*}{p_i}, \frac{q_i}{p_i} \right) \) discussed are reasonable, with properties assumed constant at initial values.

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Nomenclature*

\[
\begin{align*}
C & = \text{constant in Equation 1} \\
\dot{c} & = \text{gas compressibility, } \frac{1}{\text{psi}} \\
E_i & = \text{the } E_i \text{ function} \\
k & = \text{permeability, md} \\
K & = \text{represents } \frac{k}{\phi C} \\
k_1, k_2, k_3, k_4 & = \text{constants} \\
p & = \text{constant in Equation 1} \\
p & = \text{average reservoir pressure, psia} \\
p & = \text{pressure, psia} \\
p^{*} & = \text{flowing well pressure, psia} \\
p_i & = \text{reference pressure, psia} \\
p_i & = \text{initial pressure, psia} \\
p_i & = \text{gas flow rate in MMscf/D} \\
r & = \text{radial distance} \\
r & = \text{dimensionless radial distance} \\
t & = \text{time, hr} \\
T & = \text{dimensionless time} \\
V & = \text{temperature, °R} \\
\nu & = \text{gas compressibility factor} \\
\mu & = \text{viscosity, cp} \\
\mu_i & = \text{viscosity at 1 atm., cp} \\
\phi & = \text{porosity} \\
\phi & = \text{pseudo-pressure, } (\text{psia}^2)/\text{cp} \\
\delta & = \text{variable represents } p, p^* \text{ or } \frac{q_i}{p_i} \\
\Delta p_i & = \text{dimensionless pressure difference} \\
\psi_i & = \text{dimensionless pseudo-pressure}
\end{align*}
\]

*The units given above are required only for equations containing dimensional constants

References


