Material-Balance-Time During Linear and Radial Flow

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ABSTRACT

Production data generally consists of variable rate and variable flowing pressure. It is convenient to be able to use reservoir models that assume a constant flow rate, since these solutions have been previously derived in the well testing literature. Thus, it is necessary to have a time function capable of converting general production conditions into the equivalent constant rate solution. Blasingame¹, and later Agarwal et al² have shown that Material-Balance-Time provides an exact transformation of constant pressure data to constant rate type curves, during the boundary dominated flow regime. It also yields a reasonable approximation during radial flow, and when rate and/or pressure vary smoothly. Poe³ has investigated the effectiveness of using material-balance-time for other transient flow regimes using the constant pressure solution, rather than the constant rate solution as a base model.

The objectives of this paper are twofold. Firstly, it serves to investigate the applicability of material-balance-time during the linear flow regime (fracture flow), where the difference between the constant rate and constant pressure solutions is more pronounced. Further to this, material-balance-time correction factors are quantified for both radial and linear transient flow regimes (this has not previously been done using the constant rate solution as the base model). Secondly, it serves to illustrate by synthetic and field examples, a comparison of material-balance-time against the logarithmic superposition time function, to determine under what circumstances material-balance-time errors significantly influence rate transient interpretation, in practice.

INTRODUCTION

In pressure transient tests, the diagnostic plot (log-log plot of pressure and derivative) is an invaluable tool for reservoir characterization. For variable rate drawdown...
tests and flow and buildup tests, a time superposition function should be used to convert variable rates into an equivalent constant rate solution. Since pressure transient tests are usually dominated by infinite acting flow, the widely accepted time superposition function is one that assumes radial flow (logarithmic superposition time). One of the problems inherent in pressure transient analysis is that radial flow is not always the dominant flow regime. Thus, there is the potential for misinterpretation of the diagnostic plot in certain situations. In recent years, diagnostic plots of various different forms have been used to analyze production data. The nature of production data analysis is different than that of pressure transient analysis, primarily because of the greatly increased time scale, and because production data tends to be much noisier than pressure transient data. Most of the literature agrees that a time superposition function that assumes boundary dominated flow is more appropriate to use on a diagnostic plot of production data, than any transient superposition function, because most production data is under the influence of some sort of reservoir depletion. However, with very low permeability reservoirs, this is not necessarily the case.

Material-balance-time is the time superposition function for volumetric depletion. It is rigorous in converting variable rate production into equivalent constant rate production, provided that the flow regime is boundary dominated (volumetric depletion). Since transients are always introduced during abrupt changes in rate or flowing pressure, only a smoothly varying rate history (such as what occurs at constant bottomhole flowing pressure) is valid. Appendix A provides a theoretical proof of the validity of material-balance-time for converting boundary dominated oil production at constant pressure (smoothly varying rate) into an equivalent constant rate. Figure 1(b) shows the complete solutions for a vertical well in a cylindrical reservoir producing at constant rate and constant pressure. Figure 2(b) shows the material-balance-time corrected solution. For gas wells, the same theory holds, provided that pseudo-time is used in conjunction with material-balance-time. (pseudo-time effectively linearizes the diffusivity equation for gas by including the reservoir pressure (time) dependent compressibility and viscosity terms) In this paper, the mathematical development is limited to constant (slightly) compressible fluids.

Of interest is the applicability of material-balance-time during the infinite acting flow regime, present (and usually observable) in all production data. During infinite acting flow, material-balance-time is not a rigorous solution. Instead, a time superposition function, which follows the observed flow regime (radial, linear, bilinear etc), provides the rigorous conversion to the equivalent constant rate solution. For a rigorous superposition function, the reservoir model must be known beforehand, which is usually not the case. Accordingly, it is often impossible to apply rigorous superposition in time, in a diagnostic plot. In practice, a time superposition function, which follows the “dominant” flow regime (logarithmic for radial, square root for linear, 4th root for bilinear, material-balance-time for volumetric, etc.), is assumed. In the analysis of production data, the superposition function of choice is material-balance-time, because the emphasis is on depletion of the reservoir. Thus, it is of particular interest to determine the significance of errors that result from the universal application of material-balance-time, compared to other superposition time functions, and how they influence effective interpretation of the diagnostic plot.

THEORETICAL DEVELOPMENT

In the following sections, two flow regimes are considered, namely a) linear (fracture) flow and b) radial flow, assuming a slightly compressible fluid in an infinite reservoir.

For each flow regime, the constant rate and constant pressure solutions are compared (Figures 1(a) and 1(b). Additionally, the material-balance-time function is derived and the constant pressure solution is plotted against it (Figures 2(a) and 2(b)

**Linear Flow (Fractured well in an infinite reservoir)**

The constant rate solution for pure linear flow is as follows:
$p_D = \sqrt{D}$  ...................................................(1)

$q_D = 1/ p_D = \frac{1}{\sqrt{D}}$  ...................................................(2)

The constant pressure solution is:

$q_D = \frac{2}{\sqrt{D}}$  ...................................................(3)

For any given $q_{Dv}$ the dimensionless time corresponding to the constant pressure solution is defined as $t_{Dp}$. Similarly, the corresponding time for the constant rate solution is $t_{Dv}$. The ratio of these two times at a given $q_{Dv}$ is obtained by drawing a horizontal line, as shown in Figure 1(a). This ratio is quantified by solving (2) and (3) simultaneously:

$q_D = \frac{1}{\sqrt{Dv}} = \frac{2}{\sqrt{Dp}}$  ...................................................(4)

from which $t_{Dv}$ can be stated as a function of $t_{Dp}$

$t_{Dv} = \frac{1}{4} t_{Dp} = 2.46 t_{Dp}$  ...................................................(5)

Equation (5) shows that the correction required to convert the constant pressure solution to an equivalent constant rate is to multiply the measured time by 2.46. This conversion is exact, but applies only during pure linear flow.

The dimensionless cumulative production for the constant pressure, during linear flow is:

$Q_0 = \int_0^\frac{2}{\sqrt{Dp}} dt_{Dv} = \frac{4\sqrt{Dp}}{\sqrt{D}}$  ...................................................(6)

Thus, the material-balance-time is:

$t_{Dmb} = \frac{Q_0}{q_D} = 2 t_{Dp}$  ...................................................(7)

Equation (7) indicates that material-balance-time is double the actual time during pure linear flow. Substituting (7) into (5), we get:

$t_{Dv} = \frac{t_{Dmb}^2}{8} = 1.23 t_{Dmb}$  ...................................................(8)

Equation (8) shows that the correction required to convert the constant pressure solution to an equivalent constant rate is to multiply the material-balance-time by 1.23 (Figure 2(a)).

The above derivation indicates that a diagnostic plot that uses material-balance-time (for constant pressure linear flow) reduces the time shift from (146 % to 23 %), but does not completely eliminate it. However, it can be inferred from Figure 2(a) that unless the production data being analyzed has very high resolution, the difference between the two solutions would certainly be considered insignificant.

**Radial Flow (Vertical well in an infinite reservoir)**

The constant rate and constant pressure solutions for pure radial flow are significantly more complex than those for linear flow. They exist in analytical form (developed by Van Everdingen and Hurst) only in Laplace space, and cannot easily be analytically inverted to the time domain. Edwarson provides a numerical curve for each of the solutions, as follows:

**Constant Rate:**

\[
P_D = \frac{370.529 t_{Dv}^{0.5} + 137.582 t_{Dv} + 5.69549 t_{Dv}^{0.5}}{328.834 + 265.488 t_{Dv}^{0.5} + 45.2157 t_{Dv} + t_{Dv}^{0.5}}
\]

\[t_{Dv} < 200\]

\[
P_D = \frac{\log(t_{Dv}) + 0.40454}{2} + \frac{1}{2 t_{Dv}^{0.5}} + \frac{1}{4 t_{Dv}}
\]

\[t_{Dv} > 200\]  ...................................................(9)

**Constant Pressure:**

\[
q_D = \frac{26.7544 + 23.5377 t_{Dv}^{0.5} + 13.3813 t_{Dv} + 0.492949 t_{Dv}^{0.5}}{47.421 t_{Dv}^{0.5} + 35.5372 t_{Dv} + 2.60967 t_{Dv}^{0.5}}
\]

\[t_{Dv} < 200\]
\[
q_D = \frac{3.90086 + 2.02623t_{DP} \ln(t_{DP})}{t_D \ln(t_{DP})}
\]

For any given \(q_D\), the ratio of \(t_D\) to \(t_{DP}\) can be determined by solving the (9) and (10) simultaneously. Unlike the linear flow case, the ratio does not remain constant for radial flow, but approaches unity as time increases (see inset of Figure 1(b)).

For very small values of \(t_D\), the ratio of \(t_{Dr}\) to \(t_{DP}\) approaches that of linear flow (2.46). At a \(t_D\) of 25, which is the approximate start of practical logarithmic radial flow, the ratio is about 1.6 (60 % correction). Although this seems significant, Figure 1(b) shows that graphically, the two solutions are practically indistinguishable for values of \(t_D\) larger than 25.

As Figure 2(b) indicates, the use of material-balance-time further reduces the ratio (at \(t_D = 25\)) to about 1.17 (17 % correction). Again, the magnitude of these correction factors can be misleading, as Figure 2(b) shows that the constant rate and material-balance-time corrected constant pressure solutions are essentially indistinguishable for all values of \(t_D\).

**CASE STUDIES**

Mathematically, we have shown that there is very little observable difference between a diagnostic plot that uses material-balance-time, and the constant rate solution. Indeed, during boundary dominated flow, the transformation is exact. It is important to note that this mathematical development inherently assumes smoothly varying rate and constant pressure conditions. With real data, this assumption is commonly violated. Thus, the case studies will investigate both ideal and non-ideal operating conditions (discontinuous rate / pressure profiles). The objective is to determine to what extent (and under what conditions) the use of material-balance-time negatively influences the interpretation of the diagnostic plot. An additional objective is to compare the results to those obtained using a diagnostic plot generated using logarithmic superposition time. Here, we present both synthetic and field examples.

**Synthetic Data Examples**

For each synthetic example, two cases are investigated:

A) Smoothly varying rate profile and

B) Discontinuous rate profile

For all cases, the following parameters are used; gas reservoir (hence pseudo-pressure (\(\frac{p}{Y}\)) is used instead of pressure):

\(p_i = 20,000 \text{ kPa}\)

\(h = 10 \text{ m}\)

\(\text{porosity} = 20\%\)

\(s_w = 0\%\)

For each of these cases, a comparison has been generated between a diagnostic plot created using the logarithmic superposition time function and one created using material-balance-time. The diagnostic plots are presented in normalized rate format (\(\frac{q}{\frac{p}{Y}}\)). The derivative on the plots is simply the inverse of the standard pressure derivative used in welltest analysis.

**Vertical Well in Infinite Reservoir**

\(k = 1 \text{ mD}\)

\(s = 0\)

The synthetically generated data from this case is representative of pure radial flow. The diagnostic plots using logarithmic superposition time and material-balance-time are shown in Figures 3a (smooth rate decline) and 3b (discontinuous rates), compared against the constant rate solution.

It is clear from Figures 3a and 3b that there is practically no difference between the diagnostic plots produced from the two time functions when production variation is smooth.

Both diagnostic plots show a deviation in their derivatives from the true solution, in early-time. This is a data-averaging phenomenon that arises from not using small time-steps at the beginning of the flow period.
For the discontinuous rate case (Figure 3(b)), the diagnostic plot derived from material-balance-time shows a false boundary dominated trend developing in the late-time. The logarithmic superposition diagnostic plot correctly indicates radial flow for the duration of the production period.

**Hydraulically Fractured Well in Infinite Reservoir**

\[ k = 0.05 \text{ mD} \]
\[ k_p \omega = 1000 \text{ mD.m} \]
\[ x_f = 100 \text{ m} \]

This example exhibits the formation linear flow period of a hydraulically fractured well. The diagnostic plots are shown in Figures 4a(i), 4a(ii), 4b(i) and 4b(ii).

The smooth rate case (a) clearly indicates that both diagnostic plots (4a(i) and 4a(ii)) provide the correct flow regime identification. However, as expected, the diagnostic plot using material-balance-time is offset by a factor of about 20%. The diagnostic plot using logarithmic superposition time is also offset (this offset could be corrected by using square-root time).

The discontinuous rate profile shows significant problems with the diagnostic plot using material-balance-time (4b(i)). Both the normalized rate and derivative data have a negative unit slope, suggesting pure boundary flow, which is of course false. The superposition time diagnostic plot (4b(ii)) has little diagnostic value, as no clear interpretation is evident.

**Hydraulically Fractured Well in Bounded Reservoir**

\[ k = 1 \text{ mD} \]
\[ k_p \omega = 1000 \text{ mD.m} \]
\[ x_f = 50 \text{ m} \]
\[ r_c = 200 \text{ m} \]

This case provides production data that should exhibit, in sequence, fracture flow, pseudo-radial flow, followed by boundary dominated flow (volumetric depletion). The diagnostic plots are shown in Figures 5a(i), 5a(ii), 5b(i) and 5b(ii).

This case shows that under smoothly varying conditions, the material-balance-time diagnostic plot (5a(i)) accurately predicts the onset of boundary dominated flow. Although not shown, the plot will also yield the correct value of OGIP (Original Gas-in-Place), based on matching to a constant rate typecurve. The logarithmic superposition time diagnostic plot, however, does not correctly predict the onset of boundary dominated flow, and in fact, suggests a much larger drainage area than is actually present. Both plots match the early-time transients reasonably well (each is subjected to the same early-time averaging error previously mentioned).

Figures 5b(i) and 5b(ii) show that under abrupt rate changes, both diagnostic plots exhibit a reasonable match of \( q/\omega \). However, the material-balance-time plot derivative (5b(i)) suggests pre-mature boundary flow, while the logarithmic superposition time plot derivative shows the onset of boundary dominated flow occurring too late.

**Field Examples**

There are many examples of real production data that exhibit both smooth and discontinuous variations in rate and flowing pressure profiles. The discontinuities can occur with a wide range of frequencies and amplitudes. For example, high frequency rate changes (production “noise”) may be caused by wellbore and/or surface dynamics. In contrast, low frequency “step” changes may occur as a result of mechanical changes to the well, or due to a shift in back pressure (for example, implementing compression).

The data are analyzed using an Agarwal-Gardner diagnostic plot\(^2\), which matches the normalized rate and (inverse) pressure derivative (smoothed using the pressure integral method) to constant rate typecurves based on dimensionless time. The diagnostic plot uses material balance pseudo-time. To test the validity of the diagnostic plot, the data are also history matched using an analytical model with rigorous superposition.

**Example 1: Smoothly Varying Production Data with Step Change (long term production)**

Example 1 is a hydraulically fractured well with approximately 1.5 years of smoothly declining flow rates and bottomhole pressures, but with a fairly abrupt shift in rate near the end of the first year. Figure 6a shows the
production / pressure history for the well. Figure 6b is the diagnostic plot and Figure 6c is the history match using the analytical model.

The diagnostic plot shows a reasonable match to one of the fracture typecurves, with a transition into boundary dominated flow. The typecurve match indicates a fracture half-length in the order of 45 meters (assuming infinite conductivity). The optimum history match is obtained using a fracture half-length of 60 meters. The 25% difference is likely a result of the material-balance-time error discussed in the theory section. Both analyses yield an OGIP of approximately $75 \times 10^6 m^3$, indicating that the diagnostic plot appears to predict the onset of boundary dominated flow correctly.

**Example 2: Smoothly Varying Production Data with Step Changes (short term production)**

Example 2 contains hourly production and pressure data for a period of approximately 2 months, during which there are several step changes in pressure. The well is not hydraulically fractured. Figure 7a shows the production history. Figure 7b is the diagnostic plot. Figure 7c is the model history match.

The diagnostic plot indicates radial flow, with a slightly negative skin, followed by a transition into boundary dominated flow in the late-time. The OGIP is estimated to be in the order of $40 \times 10^6 m^3$. The model history match agrees very well with the diagnostic plot, indicating that the severity of the rate discontinuities in this case is not enough to cause potential for misinterpretation. It should be noted that the late-time data does not show a well developed boundary dominated trend (this is evident on the diagnostic plot). Consequently, the model history match is non-unique in determining OGIP. Nevertheless, the model (rectangular single layer) requires the presence of at least three boundaries to adequately match the late-time pressure behavior. It is also interesting to note that a diagnostic plot using logarithmic superposition time (not shown) shows a similar early time radial flow period, followed by a transition period in the late-time. However, the transition period observed on the logarithmic superposition plot suggests a much larger (2 times) minimum OGIP (a PSS (pseudo-steady-state) slope of 1 on the derivative plot is not quite reached on the logarithmic superposition diagnostic plot).

For this case, it appears that the diagnostic plot created using material-balance-time is sufficiently suited for approximate reservoir characterization and proper identification of flow regimes.

**Example 3: Noisy Production Data (short production)**

Example 3 has five months of daily production and pressure data. The well has been hydraulically fractured. The operational conditions for this well are such that there is a great deal of noise (high frequency and amplitude rate changes). This example could be considered the field equivalent of synthetic case 2b (or 3b). The input and results are shown in Figures 8a, 8b and 8c.

The diagnostic plot (8b) indicates fracture linear flow, followed by a late-time transition to boundary dominated flow. The plot suggests a fairly well developed boundary dominated trend with an OGIP in the order of $70 \times 10^6 m^3$. However, the pressure history match from the analytical model (8c) shows that a satisfactory match can be obtained assuming an unbounded reservoir. The transient analysis ($k$ and $x_i$) obtained from the diagnostic plot agrees reasonably well with the pressure history match. As expected, the logarithmic superposition diagnostic plot (not shown) also indicates infinite acting flow for the entire production period.

**DISCUSSION OF RESULTS**

Overall, the study has shown that a diagnostic plot that uses material-balance-time is usually adequate for flow regime identification and reservoir characterization, even early in the production life, when boundary flow has not been reached. The comparison of constant pressure (using material-balance-time) and constant rate solutions indicates that the expected error is usually minimal. More importantly, the solutions are seen to converge with time. (For instance, a hydraulically fractured well may have a 23% error during early-time linear flow that diminishes significantly during pseudo-radial flow, and finally converges to the true solution during boundary dominated flow). The mathematical comparisons are only valid for
constant bottomhole pressure conditions (smoothly varying rates), but also have practical application for smoothly declining rates and pressures, and step rate changes.

Real production data is rarely without some degree of discontinuity (whether its high frequency noise or occasional step changes in rate/pressure). The synthetic and field examples have shown that severe fluctuations in rate/pressure can present the possibility for misinterpretation of flow regimes. This is in addition to the existing transient error associated with material-balance-time. The most severe examples are caused by a combination of the following two conditions:

- High frequency and high amplitude noise
- Low permeability reservoir / short producing time

These conditions cause the diagnostic plot to show a pre-mature boundary dominated trend. Thus, the material-balance-time diagnostic plot will almost always be conservative in its estimation of drainage area and gas-in-place.

A diagnostic plot using logarithmic superposition time also provides significant potential for misinterpretation of flow regimes. It tends to overestimate the time to transition into boundary dominated flow, even when discontinuities in the rate profile are absent. Indeed, this type of plot is far better suited for analyzing infinite acting flow.

CONCLUSIONS

1) A diagnostic plot that uses material-balance-time can be used with confidence in identification of flow regimes, provided that the rate and pressure variation is smooth with time.

2) The above mentioned diagnostic plot (smoothly varying rate) can also be used to quantify reservoir properties (permeability and skin), but has early-time errors associated with it. In most cases, the magnitude of the errors will be negligible in comparison to the resolution of the production data. Nevertheless, to obtain a more accurate and complete reservoir characterization, the analyst should use a model with rigorous time superposition, to refine the estimates obtained from the diagnostic plot. (The diagnostic plot is sufficient for obtaining a rough approximation of transient parameters.) The calculation of fluids-in-place is rigorous for the diagnostic plot.

3) There is the potential for misinterpretation of flow regimes when abrupt rate / pressure fluctuations occur in combination with transient data, if material-balance-time is used. Under these conditions, the analyst should have access to both diagnostic plots (material-balance-time based and logarithmic superposition time based). The material-balance-time plot will tend to under-predict the time of the onset of boundary dominated flow, while the logarithmic superposition time plot will tend to overpredict it.

4) For production data that has severe rate / pressure fluctuations, neither the logarithmic superposition time plot nor the material-balance-time plot provide meaningful interpretations of either reservoir parameters or flow regime identification.

NOMENCLATURE

\( h = \text{net pay (m)} \)
\( k = \text{permeability (mD)} \)
\( k_{fw} = \text{fracture conductivity*width (mD.m)} \)
\( p_i = \text{initial shut-in pressure (kPa)} \)
\( q_D = \text{dimensionless rate} \)
\( Q_D = \text{dimensionless cumulative production} \)
\( r_e = \text{reservoir radius (m)} \)
\( r_{wa} = \text{apparent wellbore radius (m)} \)
\( s = \text{wellbore skin} \)
\( t_a = \text{pseudo-time (days, hours)} \)
\( t_D = \text{dimensionless time} \)
\( t_{Da} = \text{dimensionless time based on area} \)
\( t_{Dp} = \text{dimensionless time from constant pressure solution} \)
\( t_{Dr} = \text{dimensionless time from constant rate solution} \)
\( t_{mb} = \text{dimensionless material-balance-time} \)
\( t_{mbDA} = \text{dimensionless material-balance-time based on area} \)
\( t_{mb} = \text{material-balance-time (days, hours)} \)
\( x_f = \text{fracture half-length (m)} \)
REFERENCES


APPENDIX A

The following derivation proves that material-balance-time is a rigorous superposition function for boundary dominated flow:

The constant rate solution for a vertical well in a cylindrical reservoir, during pseudo-steady state is as follows:

\[ p_D = 2p_{DA} + \ln \frac{r_e}{r_{wa}} \frac{3}{4} \quad t_{DA} > 0.1 \]

where

\[ p_D = \frac{kh(p_e - p_{sw})}{141.2B[h]} \quad \text{and} \]

\[ t_{DA} = \frac{0.00634kt}{kC_a} \]

The constant pressure solution for the same conditions as above is as follows:

\[ q_D = \frac{1}{\ln \frac{r_e}{r_{wa}} \frac{3}{4}} e^{-2p_{DA}} \quad t_{DA} > 0.1 \]

The cumulative production is calculated as follows:

\[ Q_D = \frac{1}{\ln \frac{r_e}{r_{wa}} \frac{3}{4}} e^{-2p_{DA}} \int \frac{-2t}{r_e} e^{\frac{2}{r_e}} \frac{3}{r_{wa}} \frac{3}{4} dtd_{DA} \]

Where,

\[ t_{DA} = \frac{t_D}{2D_{da}^2} \]

Material-balance-time is now calculated as follows:

\[ t_{mbD} = \frac{Q_D}{q_D} = \frac{r_e^2}{2} \ln \frac{r_e}{r_{wa}} \frac{3}{4} \]

Rearranging the above, to solve for \( t_D \), we get

\[ t_D = \frac{2}{2D_{da}^2} \ln \frac{r_e}{r_{wa}} \frac{3}{4} + 1 \]

Thus,

\[ t_{DA} = \frac{2}{2D_{da}^2} \ln \frac{r_e}{r_{wa}} \frac{3}{4} + 1 \]

Substituting the above, back into the constant pressure equation, we get

\[ q_D = \frac{1}{\ln \frac{r_e}{r_{wa}} \frac{3}{4}} e^{-2p_{DA}} \]

\[ = \frac{1}{\ln \frac{r_e}{r_{wa}} \frac{3}{4}} e^{-2p_{DA}} \]

\[ = \frac{1}{\ln \frac{r_e}{r_{wa}} \frac{3}{4}} e^{-2p_{DA}} \]
The inverse of the above is \( p_D \):

\[
p_D = 2\sqrt{\frac{\kappa}{\mu}} + \ln\left(\frac{r_c}{r_w}\right) \frac{3}{4}
\]

Thus, we prove that constant rate and constant pressure are equivalent under boundary dominated conditions, when material-balance-time is used.
Figure 1a: Comparison of Constant Pressure and Constant Rate Solutions
Fracture Linear Flow

Ratio of \( t_{Dp} \) to \( t_{Dmb} \) is constant with time (2.46)

Constant Rate

Constant Pressure

Figure 1b: Comparison of Constant Pressure and Constant Rate Solutions
Cylindrical Reservoir with Vertical Well in Center

Ratio of \( t_{Dp} \) to \( t_{Dmb} \)

ratio = 1.6

Figure 2a: Comparison of Constant Pressure (Material Balance Time Corrected) and Constant Rate Solutions
Fracture Linear Flow

Ratio of \( t_{Dp} \) to \( t_{Dmb} \) is constant with time (1.23)

Constant Rate

Constant Pressure

Figure 2b: Comparison of Constant Pressure (Material Balance Time Corrected) and Constant Rate Solutions
Cylindrical Reservoir with Vertical Well in Center

Ratio of \( t_{Dp} \) to \( t_{Dmb} \)

Ratio = 1.17 (\( t_{D} = 25 \))

Single line during boundary dominated flow
Figure 3a: Case 1a- Vertical Well in Infinite Reservoir - Smooth Rate Profile (constant pressure production)
Diagnostic Plots Using Radial Superposition Time and Material Balance Time

Figure 3b: Case 1b- Vertical Well in Infinite Reservoir - Discontinuous Rate Profile
Diagnostic Plots Using Radial Superposition Time and Material Balance Time

Early-time error is due to averaging of q (rate) over the first time-step

Using mbt causes false boundary dominated flow trend in the late time

Figure 4a(i): Case 2a- Fractured Well in Infinite Reservoir - Smooth Rate Profile (constant pressure production)
Diagnostic Plot Using Material Balance Time

Figure 4a(ii): Case 2a- Fractured Well in Infinite Reservoir - Smooth Rate Profile (constant pressure production)
Diagnostic Plots Using Radial Superposition Time

xf = 80

xf = 140 m
Figure 4b(i): Case 2b- Fractured Well in Infinite Reservoir - Discontinuous Rate Profile
Diagnostic Plot Using Material Balance Time

Using mbt causes false boundary dominated flow trend; linear flow not evident from analysis.

Figure 4b(ii): Case 2b- Fractured Well in Infinite Reservoir - Discontinuous Rate Profile
Diagnostic Plots Using Radial Superposition Time

Radial superposition time does not yield a useful diagnostic plot.

Figure 5a(i): Case 3a- Fractured Well in Bounded Reservoir - Smooth Rate Profile (constant pressure production)
Diagnostic Plot Using Material Balance Time

Mbt diagnostic plot correctly predicts the onset of boundary dominated flow.

Figure 5a(ii): Case 3a- Fractured Well in Bounded Reservoir - Smooth Rate Profile (constant pressure production)
Diagnostic Plot Using Radial Superposition Time

Radial superposition time diagnostic plot overpredicts the time of transition to boundary.
Figure 5b(i): Case 3b- Fractured Well in Bounded Reservoir - Discontinuous Rate Profile
Diagnostic Plot Using Material Balance Time

Material balance time

Material balance diagnostic plot does not have enough character to identify flow regimes; however, the q/dp data matches the constant rate case very well.

Figure 5b(ii): Case 3b- Fractured Well in Bounded Reservoir - Discontinuous Rate Profile
Diagnostic Plot Using Radial Superposition Time

Superposition time

Radial superposition time diagnostic plot overpredicts the time of transition to boundary dominated flow; early-time data matches constant rate case very well.

Figure 6a: Field Example 1- Production History

Figure 6b: Field Example 1- Diagnostic Plot (Hydraulic Fracture Typecurves)

Legend
- Pressure
- Net Production

Agreement Gardner Rate vs. Time Typecurve Analysis

k = 0.25 mD
x_f = 47 m
OGIP = 79 10^6 m^3
Figure 6c: Field Example 1 - Model History Match

Figure 7a: Field Example 2 - Production History

**Figure 7b: Field Example 2: Diagnostic Plot (Radial Flow Typecurves)**

- **k** = 0.23 mD
- **xf** = 60 m
- **OGIP** = 75 $10^6$ m$^3$

- **k** = 0.36 mD
- **s** = -1.7
- **OGIP** = 38 $10^6$ m$^3$

**Figure 7c: Field Example 2 - Model History Match**

- **k** = 0.35 mD
- **s** = -2.5
- **OGIP** = 40 $10^6$ m$^3$
$k = 0.21 \text{ mD} \quad x_i = 153 \text{ m}$

$OGIP = 69 \times 10^6 \text{ m}^3$

$k = 0.17 \text{ mD} \quad x_i = 152 \text{ m}$

Unbounded Reservoir