Comparison of Pressure Transient Behaviour of Composite and Multi-Layer Reservoirs

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ABSTRACT

In pressure transient analysis, often the geological model is not known, or is ambiguous. Many well tests can be analyzed using a composite reservoir model which assumes that the flow capacity \((k^*h)\) near the wellbore is different from that away from the wellbore. There are many naturally occurring reservoirs that can legitimately be modeled this way because the transmissivity is indeed varying laterally. However there are many more reservoirs which have a different flow capacity near the well as compared to the bulk of the formation, not because of lateral permeability changes but because of layering. In these situations, it is the net pay and reserves, not the necessary the permeability, that is changing.

This paper compares the pressure transient behavior of a multi-layer system with that of a composite system, and illustrates the similarities and the differences in their respective Derivative signatures. It also investigates the extension of these two different models to pseudo-steady state forecasting. Even though the behavior of these two systems may be similar during transient flow (this is the time domain of most well tests), the long-term performance is significantly different if the improper model is used (this is the time domain of many engineering and economic decisions). The role of the geological model in well testing and deliverability forecasting is discussed, as it can have a significant effect on some parts of the analysis and an insignificant effect on other parts.

INTRODUCTION

Multi-Layer System

It is commonly known that if two or more layers of a reservoir are open to a wellbore, and are initially at a common pressure and constant drawdown, the general characteristics of the drawdown response is similar to that of a well producing from a single layer reservoir\(^{(1,2)}\). Specifically, each layer will contribute production which is proportional to its transmissibility \((kh/\mu)\). Therefore, the semi-log slope \((m)\) calculated from the infinite acting flow \(^1\) data will be nearly proportional to the sum of the individual layers transmissibilities (any modification of this slope would be due to unequal diffusivities or skin factors). For example, if a well is producing commingled at the wellbore from a layer of 2 m and a layer of 3 m net pay (both with a permeability of 20 mD), the semi-log analysis of the radial flow data (refer to Figure #1) will show a flow capacity of 100 mD.m

\(^1\) "Infinite acting flow" data will hereafter be referred to as "Radial flow" data.
If one layer of a two-layer model is limited in drainage area, a depletion of the limited layer will eventually occur. Upon depletion of this layer, a semi-log straight line with a slope "m” that is inversely proportional to the transmissibility of the more extensive layer may be observed. On the derivative analysis, after the initial radial flow, a unit slope develops followed by a transition to a second radial flow characterized by the typical zero slope. Figure #2 shows the semi-log plot for a two-layer reservoir where the extensive layer has a net pay of 2 m and a permeability of 20 mD (the corresponding dimensionless typecurve is embedded within the figure). The calculated flow capacity during the second radial flow period is 40 mD.m.

**Composite System**

Generally, the classical composite systems is represented by a circular drainage area (of constant net pay), with two concentric zones of different permeability. As with the layered system, a reservoir with a radial discontinuity (i.e., a permeability change) will also yield two distinct radial flow periods. However, the slope "m” of the initial radial flow data on the semi-log drawdown curve will only represent the transmissivity of the region of pay prior to the discontinuity located some distance (r) from the center of well. For example, if a well is producing from a net pay of 5 m with a near wellbore permeability of 20 mD, an initial flow capacity of 100 mD.m is calculated from the semi-log analysis (refer to Figure #3).

When the pressure transient reaches the outer region (of different permeability), the initial semi-log straight line will begin a transition (the shape and duration of which are dependent upon the storativity/transmissibility ratio of the two regions). Once the produced fluid is provided by the outside region, a second radial flow regime develops with a slope inversely proportional to the transmissivity of the outer zone. If the permeability in the example above decreases to 8 mD at some distance from the wellbore, a flow capacity of 40 mD.m is calculated from the late-time semi-log analysis (assuming a constant net pay of 5m). Figure #4 shows the semi-log plot and the dimensionless typecurve signature for a composite system where the reservoir flow capacity decreases from 100 mD.m to 40 mD.m away from the wellbore with a constant net pay of 5 m.

Incidentally, the composite reservoir model just described has flow capacities equivalent to the 2-layer reservoir model previously described with net pays of 2 and 3 m, and a permeability of 20 mD.

**MODEL COMPARISON**

**Comparison of Transient Gas Forecasts**

As illustrated in the previous section, composite and layered systems can have equivalent flow capacities (kh) throughout the reservoir. This implies that in a well test, if a "near wellbore kh" and a "bulk formation kh" are interpreted, it may not be possible to definitively say whether the reservoir is composite or multi-layer. However, the choice of reservoir model could have a significant impact on the long term performance of the well. This effect is assessed by calculating and comparing the deliverability predicted by each model.

To compare the deliverability from each model, equivalent composite and layered reservoir models were generated in which the flow capacity decreases from 100 mD.m to 40 mD.m @ 150m away from the wellbore, refer to Table 1, for model parameters, and to Figure 6a and 6b). Figure #5 shows a forecast comparison for the two models, which indicated that the composite system predicted a higher deliverability and cumulative production than the multi-layered model although, both models assumed a drainage radius of 150m for the limited zone and 1200m for the extensive zone.

A review of the reservoir parameters used in the forecasts indicated that the difference between the two deliverability profiles was attributed to the magnitude of the reserves contained in each of the models: the composite model, as described in the classical well testing literature, assumes a constant net pay (5 m) for the entire drainage area, while the multi-layer system incorporates a decreasing net pay for the same area. A simple volumetric calculation of the reserves contained in the multi-layer system showed an initial GIP of 324 10^6 m^3, whereas the composite system indicated an initial GIP of 791 10^6 m^3. Table 1 shows the data used in the calculations. This significant difference in reserves is due to the fact that even though the two models have the same early-time and late-time kh product (and are therefore indistinguishable), the difference has been attributed to a change in permeability for the composite (classical) model, as opposed to a change in thickness for the multi-layer model.

In order to avoid the misrepresentation of reserves, and to make a proper comparison between the two models, the composite system was modified so that it would have the same equivalent flow capacity and reserves as the layered system. Accordingly, a second scenario – the Modified Composite Model was generated by assuming a change in the net pay of the composite model rather than a change in the permeability. Its performance was then compared to that of the 2-layer model. Figure #6c and 6a show the configuration and Table 1 gives the reservoir characteristics of the two models. A simple inspection indicates that both systems will have a flow capacity of 100 mD.m near the wellbore (zone 1), and 40 mD.m in the outer reservoir (zone 2). Furthermore, both systems also have comparable reserves of about 324 10^6 m^3 (refer to Table 1 details).

Because both models (Fig 6a and 6c) have a similar kh distribution, and similar reserves, it was expected that their deliverability performance would be similar. However, as can be seen in Figure 7, this was not the case. Unexpectedly, the deliverability profile for the
composite system was again different from the equivalent layered system; specifically, the modified composite system sustained a higher deliverability in the first few months of production. It appeared that the reserves from the modified composite system were produced faster than the layered system (as explained later, this is because the higher permeability inner zone acts like a negative skin to the outer zone). The (modified) composite system's rate eventually declines to a rate lower than that of the multi-layered reservoir. The faster decline in rate is explained by the rapid depletion in reserves caused by the cumulative production. Ultimately, of course, if the flow duration is long enough, the cumulative production of both systems will be the same. However, at any given time, the cumulative production from the composite (modified) system exceeds that from the multi-layered model.

To investigate the effect of reserves size, the deliverability profile of the two systems were compared assuming unlimited reserves (ie. an unbounded reservoir). As shown in the Figure #8, the modified composite system has a significantly higher deliverability profile than the layered system. This higher rate is maintained continuously (unlike the limited reserves case shown in Figure 7), which confirms that for the models shown in Figure 7, the only reason that the rate of the modified composite system becomes lower than that of the multi-layer is that the reserves are depleted faster because of the higher rates sustained earlier.

Furthermore, if both systems are produced at a constant rate (as opposed to a constant pressure), a review of the pressure profile at the wellbore (Figure #9), indicates that the composite system requires a smaller pressure drop between the reservoir and wellbore to produce the equivalent gas rate as the multi-layered system. This too is related to the fact that the inner zone of the modified composite acts like a negative skin.

**COMPARISON OF PRESSURE BEHAVIOR**

To investigate the dissimilarities between the pressure drop and productivity between composite and multi-layered systems, the respective flow equations for the two systems are compared (refer to Appendix A). Since it is the long-term deliverability that is of greatest interest, only the steady-state equations for the two models were considered in the analysis (for simplicity's sake). By comparing the flow equations of the two systems at a constant drawdown rate, it is shown that the ratio of the pressure drop of the two systems is a function of the kh distribution of the composite system, as well as a function of the distance to the radial discontinuity (r1).

Specifically, if the higher flow capacity (kh) is near the wellbore, the analysis indicates that the composite system would require a pressure drop that is a fraction of that required for layered system (assuming no skin and equal, constant rate). Furthermore, as the magnitude of the near wellbore flow capacity increases relative to the outer zone, the smaller is the pressure drop of the composite system compared to the layered system. This implies that the composite system will produce at a greater rate than the layered system if the drawdown pressure is held constant.

Further inspection of the equations shows that if the near wellbore region is the lower flow capacity zone (which should be physically impossible for the multi-layered system, refer to Appendix A), the multi-layer model would require a lower pressure drop than the composite system, and would therefore predict a higher deliverability than the composite system at a constant drawdown pressure.

**SEMI-LOG COMPARISON**

Although in the previous section and Appendix A, it was demonstrated analytically that the pressure drop and deliverability potential of the two systems would be different, it can be more meaningful to the well tester to relate the differences via the skin effect and productivity ratio.

A semi-log comparison of the two types of composite (classical and modified) systems and the multi-layered system was generated (refer to Figure #10). As expected, the plot indicated that both composite models and the layered model exhibit equivalent pressure behavior during the initial radial flow period, but different behavior from the multi-layer model during extended production. Although the composite models exhibit the same slope "m" as the 2-layered model during the second radial flow period, the y-scale intercept of the radial flow line is different than that of the layered model, which indicates that the composite models have different skin effects than the layered model. Since the intercept of the semi-log straight line data for the composite model is lower than that for the multi-layer system, the calculated skin effect for the composite systems are lower. Thus, when the outer reservoir is the lower capacity zone, the composite models behave like a well with stimulated near wellbore region of radius r1. A late-time radial analysis of the composite system (also shown in Figure #10), indicates an apparent skin effect of about -4. If the outer reservoir is of higher capacity matrix, it is anticipated that the composite model would exhibit a positive skin effect.

Once the steady-state pressure behavior of the composite system is expressed as a function of skin, it is possible to relate the productivity of the composite model to that of the multi-layer model, using the method outlined developed by Hawkins'. However, it was necessary to modify Hawkins work to provide an relationship that would relate skin effect to varying flow capacity (as opposed to varying permeability) between
two zones of a composite system (refer to Appendix B). When the composite parameters from the example illustrated in Figure #6 were incorporated into the modified equation, a skin effect of -4.4 is calculated (which compares very well with skin effect calculated from the late-time radial analysis of the data shown in Figure #10).

For the example presented in this paper, the ideal rate represents the flow at a skin of zero (ie. the multi-layer scenario with the limited layer depleted), and the actual rate represents the flow rate at a skin other than zero (the modified composite scenario). Incorporating the calculated skin effect shows the modified composite model should exhibit about 1.8 times the rate of the multi-layer model. A review of the data presented in Figure #8 shows a final gas rate of 382 $10^4$ m$^3$ for the modified composite system which is approximately 1.7 times greater than the final gas rate of the equivalent multi-layer system.

PRACTICAL CONSIDERATIONS & FURTHER WORK

It is apparent that the composite and multi-layer models do not provide equivalent deliverability predictions due to flow patterns associated with each model. It has been demonstrated that when comparing the late-time performance of the multi-layer model (when the limited layer is depleted), to the modified composite model, both models have the same kh but the composite model displays a negative skin. This is explained by the fact that in the modified composite model, the near wellbore flow was assumed to take place in the total interval thickness whereas in the multi-layer model, the flow takes place only in the extensive layer (the other one is essentially depleted).

In other words, the near wellbore kh for the modified composite system is larger than the effective near wellbore kh for the multi-layer model. The flow paths are illustrated in Figure 11(a) for reservoirs with high, moderate, and low vertical permeability’s. In the extreme, low vertical permeability will behave like a layered reservoir.

Although open-hole logs may indicate a shale (or other barrier) between productive pay at the wellbore, crossflow within the layer may actually be present — depending upon the degree of vertical permeability. Furthermore, if the ratio of $k_x/k_z$ is very large, a composite system would effectively behave like a multi-layer system (refer to Figure #11). Consequently, this implies that if vertical permeability is incorporated into pressure transient analysis, the composite and multi-layer systems would behave similarly during steady-state flow, and that the skin effect exhibited by composite models during the steady-state flow is artificially created by the assumption that $k_x = k_z$. However, to evaluate this conclusion, further additional work is necessary.

CONCLUSIONS

The results from the analysis presented in this paper have demonstrated:

1. The modified composite and multi-layer model are equivalent models only with respect to reserves and flow capacity.
2. With respect to deliverability, the modified composite model will predict a higher deliverability during production at the same drawdown pressure, assuming that the higher flow capacity is the inner zone.
3. The additional deliverability potential of the composite model is attributed to a "negative skin effect" created by the increased flow capacity near the wellbore.
4. The negative skin effect exhibited by the modified composite model is a function of the vertical permeability.
5. The error in the deliverability forecast between the two models can be estimated using a modified Hawkin's equation to calculate the skin effect created by the composite model.

NOMENCLATURE

$k$ = permeability, md
$r$ = radius, m
$h$ = thickness of net pay, m
$\mu$ = viscosity, $\mu$ Pa.s
$t$ = time, hours
$p$ = pressure, kPa
$q$ = rate, $10^4$m$^3$/d

Subscripts and Superscripts

1 = Altered/ Near wellbore zone
2 = Outer reservoir zone
w = wellbore
s = skin

REFERENCES

3. HAWKINS, M.F., Jr., A Note of the Skin Effect; Trans, AIME, (1956) 207, 356-357.

APPENDIX A: STEADY-STATE FLOW EQUATIONS

In order to compare the pressure behavior of the composite and layered models during steady-state flow, equations for both systems must be established. The general form of Darcy's equation in radial form is:
\[ \frac{q}{2\pi r h} = - \frac{kdp}{\mu dr} \]  

.................................................. (1)

If equation (1) is solved for each zone of the modified composite system (as defined in figure #), and then solved for \( q \) as a function of \((p_e - p_w)\), the following is obtained:

\[ q = \frac{-2\pi (p_e - p_w)}{\mu \ln \frac{r_e}{r_w} + \mu \frac{1}{(kh)_2} \ln \frac{r_e}{r_i}} \]  

.................................................. (2)

*where \( c \) designates a composite system

For the layered system, it is assumed that the limited layer is depleted and that only the extensive layer \((kh)_2\) is producing. It is then easily seen that solving equation (1) can be solved as for the layered system as shown below (3)

\[ q = \frac{-2\pi (kh)_2 (p_e - p_w)}{\mu \ln \frac{r_e}{r_w}} \]  

.................................................. (3)

*where \( l \) designates a layered system

Equating equations (2) and (3) and solving for pressures the ratio of pressure gives:

\[ \frac{(p_e - p_w)_c}{(p_e - p_w)_l} = \left[ \frac{\mu \frac{1}{(kh)_1} \ln \frac{r_e}{r_w} + \mu \frac{1}{(kh)_2} \ln \frac{r_e}{r_i}}{(kh)_2} \right] \]  

The limit of equation (4) as \((kh)_1\) approaches infinities

\[ \lim_{(kh) \to \infty} \frac{(p_e - p_w)_c}{(p_e - p_w)_l} = \frac{\ln \frac{r_e}{r_i}}{\ln \frac{r_e}{r_w}} \leq 1.0 \]

.................................................. (5)

where \( r_w < r_i < r_e \)

This fact that this ratio is less than unity shows that the pressure drop of a composite model should be less than that of a layered model when the high capacity zone is near the wellbore.

Alternatively, if equation (4) is evaluated as \((kh)_2\) approaches infinity, it can be seen that:

\[ \lim_{(kh)_2 \to \infty} \frac{(p_e - p_w)_c}{(p_e - p_w)_l} = \frac{\ln \frac{r_e}{r_i} + \ln \frac{r_e}{r_w}}{\ln \frac{r_e}{r_w}} = \infty \]

It is important to note that it is not likely that a layered system will occur where the near wellbore \( kh \) is less than \((kh)_2\). This scenario would require that the flow capacity of the limited layer be negative so that the sum of the flow capacities during the first radial flow period is less than the flow capacity of the second radial flow period which is dominated by the extensive layer. Its anticipated that such a scenario could only occur if the layered system had large positive skin factors which could be treated as a low permeability matrix near the wellbore (but then the composite and layered would no longer be equivalent).

**APPENDIX B: SKIN EFFECT AND PRODUCTIVITY**

In 1956, Hawkins introduced the following equation for relating skin factor to permeability varying radially away from the wellbore:

\[ s = \left( \frac{k}{k_i} - 1 \right) \ln \left( \frac{r_e}{r_w} \right) \]  

.................................................. (1)

However, for the purposes of this paper, it is necessary to modify Hawkins equation to relate skin factor varying flow capacity. Following Hawkins procedure to obtain equation (1), the following equation relating skin factor to varying flow capacity can be obtained:

\[ s = \left[ \frac{(k_i h_i)}{k_i h_1} - 1 \right] \ln \left( \frac{r_1}{r_w} \right) \]  

.................................................. (2)

The productivity ratio of a well can then be related to skin factor by equation given below:

\[ \frac{q_s}{q_{wd}} = \frac{\ln \left( \frac{r_e}{r_w} \right)}{\ln \left( \frac{r_e}{r_w} + s \right)} \]  

.................................................. (3)
<table>
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<tr>
<th></th>
<th>2-LAYER MODEL (Fig 6a)</th>
<th>Classical Composite Model (Fig 6b)</th>
<th>Modified Composite Model (Fig 6c)</th>
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<td>Extensive Layer</td>
<td>Inner Region</td>
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Note: These 3 models have the same early-time kh (100 mD.m) and late-time kh (40 mD.m)
FIGURE #1: SEMI-LOG ANALYSIS OF AN INFINITE 2-LAYERED SYSTEM

FIGURE #2: LATE-TIME SEMI-LOG ANALYSIS OF 2-LAYERED SYSTEM
FIGURE #3: EARLY-TIME RADIAL ANALYSIS - COMPOSITE MODEL

FIGURE #4: LATE-TIME RADIAL ANALYSIS - COMPOSITE MODEL
FIGURE #5: FORECAST COMPARISON OF COMPOSITE MODEL AND LAYER MODEL

$h_1 = 5\text{m}, h_2 = 2\text{m}$
$r_1 = 150, r_2 = 1200 \text{ m}$

Layered Model | Composite Model (Classical) | Modified Composite Model

FIGURE #6: RESERVOIR CONFIGURATIONS
FIGURE #7: FORECAST COMPARISON OF MODIFIED COMPOSITE SYSTEM VS LAYERED SYSTEM

FIGURE #8: FORECAST COMPARISON OF MODIFIED COMPOSITE SYSTEM VS LAYERED SYSTEM
(Unlimited Reserves; $r_2 = \infty, r_t = 150$ m)
FIGURE #9: PRESSURE PROFILE COMPARISON

FIGURE #10: SEMI-LOG COMPARISON OF COMPOSITE SYSTEM VS LAYERED SYSTEM
FIGURE #11: MODIFIED COMPOSITE SYSTEM RESERVOIR FLOW PATTERNS